

BRST Cohomology and Nonlocal Conserved Charges

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A relation is found between nonlocal conserved charges in string theory and certain ghost-number two states in the BRST cohomology. This provides a simple proof that the nonlocal conserved charges for the superstring in an $AdS_5 \times S^5$ background are BRST-invariant in the pure spinor formalism and are κ -symmetric in the Green-Schwarz formalism.

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1. Introduction

Maldacena's conjecture that $d = 4$ $\mathcal{N} = 4$ super-Yang-Mills theory is dual to superstring theory in an $AdS_5 \times S^5$ background has been difficult to prove since the perturbative descriptions of these two theories do not overlap. To obtain non-perturbative information about the two theories, one possible tool could be integrability and there have been various papers discussing this possibility both on the super-Yang-Mills side and on the superstring side.

On the superstring side, Bena, Polchinski and Roiban [1] constructed an infinite set of nonlocal conserved charges for the Green-Schwarz (GS) superstring in an $AdS_5 \times S^5$ background², suggesting an integrable structure. Vallilo [4] then constructed an analogous set of nonlocal conserved charges using the pure spinor formalism for the superstring in an $AdS_5 \times S^5$ background [5]. These nonlocal charges for the superstring were related in [6] to a corresponding set of nonlocal charges on the super-Yang-Mills side.

In the GS formalism for the superstring, invariance under κ -transformations is crucial for determining the physical spectrum. For example, classical κ -symmetry is preserved in a curved background when the background satisfies the low-energy supergravity equations of motion, so onshell massless vertex operators must be κ -symmetric at least at the classical level. Although quantum κ -transformations and massive GS vertex operators are not yet understood, it is reasonable to expect that all physical GS states should be κ -symmetric. This would imply that the conserved charges for physical symmetries should also be κ -symmetric.

In the pure spinor formalism for the superstring, the role of κ -symmetry is replaced by BRST invariance. In this case, quantization and massive vertex operators are well-understood, and the physical spectrum is described by states in the BRST cohomology. So physical symmetries in the pure spinor formalism must be BRST-invariant.

Surprisingly, it has not been previously verified if the nonlocal conserved charges of [1] are κ -symmetric, or if the nonlocal conserved charges of [4] are BRST-invariant. This led some people (including this author) to conclude that the charges were not κ -symmetric and to question their physical significance. Due to the insistence of Witten [7] that these nonlocal charges should describe physical symmetries in analogy with the nonlocal charges

² These nonlocal charges were also independently found by Polyakov [2]. Similar nonlocal charges have been proposed in [3].

on the super-Yang-Mills side [6], the κ -symmetry and BRST invariance of the nonlocal charges of [1] and [4] were investigated.

As described in section 2, the existence of an infinite set of BRST-invariant nonlocal charges can be deduced from the absence of certain states in the BRST cohomology at ghost-number two. These ghost-number two states are $f_{AB}^C h^A h^B$ where h^A are the BRST-invariant ghost-number one states associated with the global isometries and f_{AB}^C are the structure constants. Whenever $f_{AB}^C h^A h^B$ can be written as $Q\Omega^C$ for some Ω^C (i.e. whenever $f_{AB}^C h^A h^B$ is not in the BRST cohomology), one can construct an infinite set of BRST-invariant nonlocal charges. It would be interesting to know if quantum corrections to the ghost-number two cohomology are related to the potential anomalies discussed in [8].

In section 3 of this paper, it is shown using the pure spinor formalism for the superstring in an $AdS_5 \times S^5$ background that the relevant ghost-number two states are absent from the classical BRST cohomology. The corresponding infinite set of BRST-invariant nonlocal charges is then explicitly constructed and shown to coincide with the conserved charges found by Vallilo in [4]. It is also shown that the conserved charges of Bena, Polchinski and Roiban in [1] are κ -symmetric.

2. Relation of Nonlocal Charges with BRST Cohomology

Suppose one has a BRST-invariant string theory with global physical symmetries described by the charges $q^A = \int d\sigma j^A(\sigma)$. Since these symmetries take physical states to physical states, $q^A = \int d\sigma j^A(\sigma)$ must satisfy $Q(q^A) = 0$ where Q is the BRST charge. Note that $\{Q, b_0\} = H$ where H is the Hamiltonian, so BRST invariance implies charge conservation if q^A commutes with the b_0 ghost, i.e. if q^A can be chosen in Siegel gauge. With the exception of the zero-momentum dilaton, it is expected that all ghost-number zero states in the BRST cohomology can be chosen in Siegel gauge.³

Since $Q(\int d\sigma j^A(\sigma)) = 0$, $Q(j^A) = \partial_\sigma h^A$ for some h^A of ghost-number one. And $Q^2 = 0$ implies that $Q(\partial_\sigma h^A) = 0$, which implies that $Q(h^A) = 0$ since there are no σ -independent worldsheet fields.

³ In the pure spinor formalism for the superstring, there is no natural b ghost. Nevertheless, it is expected that for any ghost-number zero state in the pure-spinor BRST cohomology, there exists a gauge in which the state is annihilated by H .

Consider the BRST-invariant ghost-number two states $f_{AB}^C : h^A h^B :$ where f_{AB}^C are the structure constants and normal-ordering is defined in any BRST-invariant manner, e.g. $: h^A(z) h^B(z) := \frac{1}{2\pi i} \oint dy (y-z)^{-1} h^A(y) h^B(z)$ where the contour of y goes around the point z . It will now be shown that whenever $f_{AB}^C : h^A h^B :$ is not in the BRST cohomology⁴, i.e. whenever there exists an operator Ω^C satisfying $Q(\Omega^C) = f_{AB}^C : h^A h^B :,$ one can construct an infinite set of nonlocal BRST-invariant charges.

To prove this claim, consider the nonlocal charge

$$k^C = f_{AB}^C : \int_{-\infty}^{\infty} d\sigma j^A(\sigma) \int_{-\infty}^{\sigma} d\sigma' j^B(\sigma') : .$$

Using $Q(j^A) = \partial_\sigma h^A$, one finds that $Q(k^C) = \int d\sigma l^C(\sigma)$ where

$$l^C = -2f_{AB}^C : h^A(\sigma) j^B(\sigma) : .$$

One can check that $Q(l^C) = f_{AB}^C \partial_\sigma (: h^A h^B :)$, so $Q(l^C - \partial_\sigma \Omega^C) = 0$ where Ω^C is the operator which is assumed to satisfy $Q(\Omega^C) = f_{AB}^C : h^A h^B :,$

Since $(l^C - \partial_\sigma \Omega^C)$ has +1 conformal weight and since BRST cohomology is only nontrivial at zero conformal weight, $l^C - \partial_\sigma \Omega^C = Q(\Sigma^C)$ for some Σ^C . Using Σ^C , one can therefore construct the nonlocal BRST-invariant charge

$$\tilde{q}^C = f_{AB}^C : \int_{-\infty}^{\infty} d\sigma j^A(\sigma) \int_{-\infty}^{\sigma} d\sigma' j^B(\sigma') : - \int_{-\infty}^{\infty} d\sigma \Sigma^C(\sigma).$$

By repeatedly commuting \tilde{q}^C with \tilde{q}^D , one generates an infinite set of nonlocal BRST-invariant charges. So as claimed, $f_{AB}^C : h^A h^B := Q(\Omega^C)$ implies the existence of an infinite set of nonlocal BRST-invariant charges.

3. BRST-Invariant Charges in $AdS_5 \times S^5$ Background

The results of the previous section will now be applied to the charges in an $AdS_5 \times S^5$ background using the pure spinor formalism for the superstring. As in the Metsaev-Tseytlin GS action in an $AdS_5 \times S^5$ background [10], the action in the pure spinor formalism [5] is constructed from left-invariant currents $J^A = (g^{-1} \partial g)^A$ where $g(x, \theta, \bar{\theta})$ takes values

⁴ This BRST cohomology is defined in the “extended” Hilbert space which includes the zero mode of the x^m variables. As explained in [9], the inclusion of the x^m zero mode in the Hilbert space allows global isometries to be described by ghost-number one states in the cohomology.

in the coset $PSU(2, 2|4)/SO(4, 1) \times SO(5)$, $A = ([ab], m, \alpha, \widehat{\alpha})$ ranges over the 30 bosonic and 32 fermionic elements in the Lie algebra of $PSU(2, 2|4)$, $[ab]$ labels the $SO(4, 1) \times SO(5)$ “Lorentz” generators, $m = 0$ to 9 labels the “translation” generators, and $\alpha, \widehat{\alpha} = 1$ to 16 label the fermionic generators. The action in the pure spinor formalism also involves left and right-moving bosonic ghosts, $(\lambda^\alpha, w_\alpha)$ and $(\widehat{\lambda}^{\widehat{\alpha}}, \widehat{w}_{\widehat{\alpha}})$, which satisfy the pure spinor constraints $\lambda\gamma^m\lambda = \widehat{\lambda}\gamma^m\widehat{\lambda} = 0$. These pure spinor ghosts transform as spinors under the local $SO(4, 1) \times SO(5)$ transformations and couple to the $AdS_5 \times S^5$ spin connection in the worldsheet action through their Lorentz currents $N_{ab} = \frac{1}{2}w\gamma_{ab}\lambda$ and $\widehat{N}_{ab} = \frac{1}{2}\widehat{w}\gamma_{ab}\widehat{\lambda}$.

To construct the nonlocal charges, the notation and conventions of [4] will be used where

$$\begin{aligned} J_0 &= (g^{-1}\partial g)^{[ab]}T_{[ab]}, & J_1 &= (g^{-1}\partial g)^\alpha T_\alpha, & J_2 &= (g^{-1}\partial g)^m T_m, \\ J_3 &= (g^{-1}\partial g)^{\widehat{\alpha}} T_{\widehat{\alpha}}, & N &= \frac{1}{2}(w\gamma^{[ab]}\lambda)T_{[ab]} \\ \overline{J}_0 &= (g^{-1}\overline{\partial}g)^{[ab]}T_{[ab]}, & \overline{J}_1 &= (g^{-1}\overline{\partial}g)^\alpha T_\alpha, & \overline{J}_2 &= (g^{-1}\overline{\partial}g)^m T_m, \\ \overline{J}_3 &= (g^{-1}\overline{\partial}g)^{\widehat{\alpha}} T_{\widehat{\alpha}}, & \widehat{N} &= \frac{1}{2}(\widehat{w}\gamma^{[ab]}\widehat{\lambda})T_{[ab]}, \\ \partial &= \frac{1}{2}(\frac{\partial}{\partial\tau} + \frac{\partial}{\partial\sigma}), & \overline{\partial} &= \frac{1}{2}(\frac{\partial}{\partial\tau} - \frac{\partial}{\partial\sigma}), \end{aligned} \quad (3.1)$$

and T_A are the $PSU(2, 2|4)$ Lie algebra generators. It will be convenient to also introduce the notation

$$\lambda = \lambda^\alpha T_\alpha, \quad \widehat{\lambda} = \widehat{\lambda}^{\widehat{\alpha}} T_{\widehat{\alpha}}.$$

Note that λ and $\widehat{\lambda}$ are fermionic since $(T_\alpha, T_{\widehat{\alpha}})$ are fermionic and $(\lambda^\alpha, \widehat{\lambda}^{\widehat{\alpha}})$ are bosonic.

Under classical BRST transformations generated by

$$Q = \int d\sigma (\lambda^\alpha J_3^{\widehat{\alpha}} + \widehat{\lambda}^{\widehat{\alpha}} \overline{J}_1^\alpha) \delta_{\alpha\widehat{\alpha}},$$

g transforms by right-multiplication as

$$Q(g) = g(\lambda + \widehat{\lambda}) \quad (3.2)$$

and the pure spinors transform as

$$Q(N) = -2[J_3, \lambda], \quad Q(\widehat{N}) = -2[\overline{J}_1, \widehat{\lambda}], \quad Q(\lambda) = Q(\widehat{\lambda}) = 0.$$

The left-invariant currents therefore transform as

$$Q(J_j) = \delta_{j+3,0}\partial\lambda + [J_{j+3}, \lambda] + \delta_{j+1,0}\partial\widehat{\lambda} + [J_{j+1}, \widehat{\lambda}],$$

$$Q(\bar{J}_j) = \delta_{j+3,0} \bar{\partial} \lambda + [\bar{J}_{j+3}, \lambda] + \delta_{j+1,0} \bar{\partial} \hat{\lambda} + [\bar{J}_{j+1}, \hat{\lambda}],$$

where j is defined modulo 4, i.e. $J_j \equiv J_{j+4}$.

To prove the existence of an infinite set of BRST-invariant charges, one needs to find an $\Omega = \Omega^C T_C$ satisfying $Q\Omega =: \{h, h\}$: where $h = h^A T_A$, $Q(j) = \partial_\sigma h$, and $q^A = \int d\sigma j^A$ are the charges associated with the global $PSU(2, 2|4)$ isometries. It will be shown at the end of subsection (3.1) that $Q(j) = \frac{1}{2} \partial_\sigma (g(\lambda - \hat{\lambda}) g^{-1})$, so

$$h = \frac{1}{2} g(\lambda - \hat{\lambda}) g^{-1}. \quad (3.3)$$

Note that h is BRST-invariant since

$$Q(h) = \frac{1}{2} g\{(\lambda + \hat{\lambda}), (\lambda - \hat{\lambda})\} g^{-1} = \frac{1}{2} g((\lambda^\alpha \gamma_{\alpha\beta}^m \lambda^\beta) T_m - (\hat{\lambda}^{\hat{\alpha}} \gamma_{\hat{\alpha}\hat{\beta}}^m \hat{\lambda}^{\hat{\beta}}) T_m) g^{-1} = 0 \quad (3.4)$$

because of the pure spinor constraint. Consider the ghost-number two state

$$\{h, h\} = \frac{1}{2} g(\lambda - \hat{\lambda})(\lambda - \hat{\lambda}) g^{-1} = -\frac{1}{2} g\{\lambda, \hat{\lambda}\} g^{-1}. \quad (3.5)$$

Since $\{\lambda + \hat{\lambda}, \lambda + \hat{\lambda}\} = 2\{\lambda, \hat{\lambda}\}$, one can write this state as $Q\Omega$ where

$$\Omega = -\frac{1}{4} g(\lambda + \hat{\lambda}) g^{-1}. \quad (3.6)$$

So $Q\Omega = \{h, h\}$, which implies the existence of an infinite set of BRST-invariant charges.

3.1. Explicit construction of BRST-invariant nonlocal charges

To explicitly construct these BRST-invariant charges, suppose one has a current whose τ -component a satisfies

$$Qa = \partial_\sigma \Lambda + [a, \Lambda] \quad (3.7)$$

for some Λ . Then the charge

$$P(e^{-\int_{-\infty}^{\infty} d\sigma a(\sigma)}) \equiv \quad (3.8)$$

$$1 - \int_{-\infty}^{\infty} d\sigma a(\sigma) + \int_{-\infty}^{\infty} d\sigma a(\sigma) \int_{-\infty}^{\sigma} d\sigma' a(\sigma') - \int_{-\infty}^{\infty} d\sigma a(\sigma) \int_{-\infty}^{\sigma} d\sigma' a(\sigma') \int_{-\infty}^{\sigma'} d\sigma'' a(\sigma'') + \dots$$

satisfies $Q(P(e^{-\int_{-\infty}^{\infty} d\sigma a(\sigma)})) = 0$. So $P(e^{-\int_{-\infty}^{\infty} d\sigma a(\sigma)})$ is a BRST-invariant charge.

To construct a satisfying (3.7), consider

$$a(c_j, \bar{c}_j) = g(c_0 N + c_1 J_1 + c_2 J_2 + c_3 J_3 + \bar{c}_0 \bar{N} + \bar{c}_1 \bar{J}_1 + \bar{c}_2 \bar{J}_2 + \bar{c}_3 \bar{J}_3) g^{-1} \quad (3.9)$$

where c_j and \bar{c}_j are constant coefficients. Note that $a(c_j, \bar{c}_j)$ is invariant under the local $SO(4, 1) \times SO(5)$ transformations.

Using the BRST transformations of (3.2),

$$\begin{aligned} Qa &= g[\lambda + \hat{\lambda}, c_0 N + \bar{c}_0 \bar{N} + \sum_{k=1}^3 (c_k J_k + \bar{c}_k \bar{J}_k)]g^{-1} \\ &\quad + g(-2c_0[J_3, \lambda] - 2\bar{c}_0[\bar{J}_1, \hat{\lambda}] + c_1 \partial \lambda + c_3 \partial \hat{\lambda} + \bar{c}_1 \bar{\partial} \lambda + \bar{c}_3 \bar{\partial} \hat{\lambda})g^{-1} \\ &\quad + g \sum_{k=1}^3 (c_k[J_{k+3}, \lambda] + c_k[\bar{J}_{k+1}, \hat{\lambda}] + \bar{c}_k[\bar{J}_{k+3}, \lambda] + \bar{c}_k[\bar{J}_{k+1}, \hat{\lambda}])g^{-1}. \end{aligned} \quad (3.10)$$

And defining

$$\Lambda(b, \bar{b}) = g(b\lambda + \bar{b}\hat{\lambda})g^{-1},$$

where b and \bar{b} are constant coefficients, one obtains

$$\begin{aligned} \partial_\sigma \Lambda + [a, \Lambda] &= g(b(\partial \lambda - \bar{\partial} \lambda) + \bar{b}(\partial \hat{\lambda} - \bar{\partial} \hat{\lambda}))g^{-1} + g[\sum_{j=0}^3 (J_j - \bar{J}_j), b\lambda + \bar{b}\hat{\lambda}]g^{-1} \\ &\quad + g[c_0 N + \bar{c}_0 \bar{N} + \sum_{k=1}^3 (c_k J_k + \bar{c}_k \bar{J}_k), b\lambda + \bar{b}\hat{\lambda}]g^{-1}. \end{aligned} \quad (3.11)$$

Setting (3.10) equal to (3.11) and using the worldsheet equations of motion [5][4]

$$\bar{\partial} \lambda + [\bar{J}_0, \lambda] = -\frac{1}{2}[\bar{N}, \lambda], \quad \partial \hat{\lambda} + [J_0, \hat{\lambda}] = -\frac{1}{2}[N, \hat{\lambda}], \quad (3.12)$$

and the pure spinor constraints $[\lambda, N] = [\hat{\lambda}, \bar{N}] = 0$, one obtains the conditions

$$c_1 = b, \quad -\bar{c}_3 = \bar{b}, \quad (3.13)$$

$$\begin{aligned} -c_1 + c_2 &= bc_1 + b, \quad -c_2 + c_3 = bc_2 + b, \quad -c_3 - 2c_0 = bc_3 + b, \\ -c_1 &= \bar{b}c_1 + \bar{b}, \quad -c_2 + c_1 = \bar{b}c_2 + \bar{b}, \quad -c_3 + c_2 = \bar{b}c_3 + \bar{b}, \quad 2c_0 + c_3 = -2\bar{b}c_0 + \bar{b}, \\ 2\bar{c}_0 + \bar{c}_1 &= -2b\bar{c}_0 - b, \quad -\bar{c}_1 + \bar{c}_2 = b\bar{c}_1 - b, \quad -\bar{c}_2 + \bar{c}_3 = b\bar{c}_2 - b, \quad -\bar{c}_3 + \bar{c}_2 = b\bar{c}_3 - b, \\ -\bar{c}_1 - 2\bar{c}_0 &= \bar{b}\bar{c}_1 - \bar{b}, \quad -\bar{c}_2 + \bar{c}_1 = \bar{b}\bar{c}_2 - \bar{b}, \quad -\bar{c}_3 + \bar{c}_2 = \bar{b}\bar{c}_3 - \bar{b}. \end{aligned}$$

The conditions of (3.13) are solved by

$$c_0 = \frac{1}{2}(1 - \mu^2), \quad c_1 = \pm \mu^{\frac{1}{2}} - 1, \quad c_2 = \mu - 1, \quad c_3 = \pm \mu^{\frac{3}{2}} - 1, \quad (3.14)$$

$$\bar{c}_0 = \frac{1}{2}(\mu^{-2} - 1), \quad \bar{c}_1 = 1 \mp \mu^{-\frac{3}{2}}, \quad \bar{c}_2 = 1 - \mu^{-1}, \quad \bar{c}_3 = 1 \mp \mu^{-\frac{1}{2}},$$

$$b = \pm \mu^{\frac{1}{2}} - 1, \quad \bar{b} = \pm \mu^{-\frac{1}{2}} - 1,$$

which coincides with the solution of [4] for conserved currents.

Note that the global charge $q = \int d\sigma j(\sigma)$ can be obtained from (3.9) by expanding $a(\mu)$ near $\mu = 1$. If $\mu = 1 + \epsilon$, one finds that $a(\mu) = \epsilon j + \mathcal{O}(\epsilon^2)$ and $\Lambda(\mu) = \epsilon h + \mathcal{O}(\epsilon^2)$ where $Q(j) = h$. So from the formulas

$$b(\mu) = \mu^{\frac{1}{2}} - 1 = \frac{1}{2}\epsilon + \mathcal{O}(\epsilon^2), \quad \bar{b}(\mu) = \mu^{-\frac{1}{2}} - 1 = -\frac{1}{2}\epsilon + \mathcal{O}(\epsilon^2),$$

one learns that

$$h = \lim_{\epsilon \rightarrow 0} \epsilon^{-1} \Lambda(\mu) = \frac{1}{2}g(\lambda - \hat{\lambda})g^{-1}, \quad (3.15)$$

as was claimed in (3.3).

3.2. κ -symmetry of nonlocal GS charges

Finally, it will be shown that the nonlocal GS conserved charges of [1] are κ -symmetric. In conformal gauge for the GS superstring, the κ -transformations of g and the κ -transformations of the left-invariant currents can be obtained from the BRST transformations of (3.2) by replacing λ with $[\kappa, J_2]$ and replacing $\hat{\lambda}$ with $[\hat{\kappa}, \bar{J}_2]$ where $\kappa = \kappa^\alpha T_\alpha$ and $\hat{\kappa} = \hat{\kappa}^\alpha T_\alpha$. This is the $AdS_5 \times S^5$ version of the procedure adopted in [11] where λ^α is replaced by $\Pi^m(\gamma_m \kappa)^\alpha$ and $\hat{\lambda}^\alpha$ is replaced by $\bar{\Pi}^m(\gamma_m \hat{\kappa})^\alpha$. In GS conformal gauge, $\eta_{mn} J_2^m J_2^n = \eta_{mn} \bar{J}_2^m \bar{J}_2^n = 0$ and the κ -transformations are constrained to satisfy $\kappa^\alpha \bar{J}_1^\alpha \delta_{\alpha\hat{\alpha}} = \hat{\kappa}^\alpha J_3^\alpha \delta_{\alpha\hat{\alpha}} = 0$ so that the h_{zz} and $h_{\bar{z}\bar{z}}$ components of the worldsheet metric do not transform. Together with the GS equations of motion $[J_2, \bar{J}_1] = 0$ and $[\bar{J}_2, J_3] = 0$, these conditions imply that

$$[\lambda, J_2] = [\hat{\lambda}, \bar{J}_2] = [\lambda, \bar{J}_1] = [\hat{\lambda}, J_3] = 0. \quad (3.16)$$

Using the current of (3.9) with $c_0 = \bar{c}_0 = 0$, one finds that (3.10) is equal to (3.11) if

$$c_1 = b, \quad -\bar{c}_3 = \bar{b}, \quad (3.17)$$

$$-c_1 + c_2 = bc_1 + b, \quad -c_3 = bc_3 + b,$$

$$-c_1 = \bar{b}c_1 + \bar{b}, \quad -c_2 + c_1 = \bar{b}c_2 + \bar{b}, \quad c_3 = \bar{b},$$

$$\begin{aligned}\bar{c}_1 &= -b, \quad -\bar{c}_2 + \bar{c}_3 = b\bar{c}_2 - b, \quad -\bar{c}_3 = b\bar{c}_3 - b, \\ -\bar{c}_1 &= \bar{b}\bar{c}_1 - \bar{b}, \quad -\bar{c}_3 + \bar{c}_2 = \bar{b}\bar{c}_3 - \bar{b}.\end{aligned}$$

The conditions of (3.17) are solved by

$$\begin{aligned}c_1 &= \pm\mu^{\frac{1}{2}} - 1, \quad c_2 = \mu - 1, \quad c_3 = \pm\mu^{-\frac{1}{2}} - 1, \\ \bar{c}_1 &= 1 \mp \mu^{\frac{1}{2}}, \quad \bar{c}_2 = 1 - \mu^{-1}, \quad \bar{c}_3 = 1 \mp \mu^{-\frac{1}{2}}, \\ b &= \pm\mu^{\frac{1}{2}} - 1, \quad \bar{b} = \pm\mu^{-\frac{1}{2}} - 1,\end{aligned}\tag{3.18}$$

which coincides with the conserved GS charges of [1].

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